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Technical Notes

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Proposed Inflow/Outflow Boundary Condition for Direct Computation of Aerodynamic Sound

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Introduction

RECENT studies of aerodynamic sound generation and radiation are relying more frequently on computer simulations. Direct simultaneous computation of the flowfield and the radiated sound offers the most detailed description of aerodynamic sound generation processes. A primary difficulty in the development of computational techniques for direct aeroacoustic computation has been the inadequacy of numerical boundary conditions. For many flows of interest, and in particular jets, the ideal domain should be infinite in all directions. Practically, it is necessary to artificially truncate the domain. The procedure of truncation must be done in such a way that the solution within the domain is invariant, to within some prescribed accuracy requirement, to the truncation location.

There is a rich literature discussing nonreflecting boundary conditions that are applicable to compressible flows. Many of the works either treat linear hyperbolic equations or, in the case of Euler or Navier–Stokes equations, linearize the equations for the formulation of boundary conditions.[1–5] Several boundary treatments are reviewed by Givoli.[6] It is possible for some linear equations and domain geometries to obtain exact boundary treatments[7]; however, most such schemes are inapplicable to shear flows. Shear flows have persistent nonlinear downstream hydrodynamics that are analytically intractable. Approximate boundary conditions that are based upon linear analysis may be applied but typically lack sufficient accuracy to preclude the reflection of high-energy flow structures passing out of the domain. The disparity of amplitudes between the flowfield vorticity and dilatation and the radiating sound field requires extreme accuracy at the boundaries for all angles of incidence. The sound field energy may be orders of magnitude smaller than the flow energy.

In cases where high-amplitude nonlinear disturbances must exit the domain with minimal reflection, it has been found necessary to add nonphysical exit zones onto the computation. This method was first proposed for direct acoustic simulation by Colonius et al.[8] and was based on previous applications in hydrodynamic computations.[9] Colonius et al. employed numerical filtering to damp disturbances in the exit zone. This was done in such a way as to minimize reflections. A different boundary zonal treatment has been proposed by Ta'asan and Nark,[10] who add a convective term to the the linear Euler equations and thereby force them to be supersonic at the borders of the numerical domain. Berenger[11] proposed another zonal boundary treatment for Maxwell's equations, which was extended to the Euler equations by Hu.[12] In this zonal method, a damping term is added to the equations that drives the solution toward a quiescent target state.

In direct turbulence simulations, it is sometimes necessary to feed turbulence into the computational box, which circumvents the expense and difficulty of simulating transition to turbulence. In a direct acoustic simulation, inflow turbulence also requires special treatment to prevent nonphysical acoustic radiation from the inflow boundary. The situation is somewhat different from that of the outflow but the problem is the same: to pass flow structures through the domain edge without generating spurious sound.

This study proposes a novel zonal approach for the inflow and outflow problems combining basic techniques proposed by Ta'asan and Nark[10] and Berenger.[11]

Proposed Boundary Zones

The full compressible Navier–Stokes equations in conservative variables are considered. We propose to add two additional terms to each of the equations that would be effective only in the inflow or outflow zones of the computation (see Fig. 1). Taking x_1 to be the streamwise direction, $U(x_1)$ an artificial convection velocity, and $\sigma(x_1)$ an artificial damping function, the equations for the entire domain with the additional terms are compactly written as

$$
\begin{aligned}
&\partial_t\rho + U(x_1)\rho_{,1} + (\rho u_j)_{,j} = -\sigma(x_1)(\rho - \rho_{\text{target}})\\
&\partial_t(\rho u_i) + U(x_1)(\rho u_i)_{,1} + (\rho u_i u_j)_{,j} = \tau_{ij,j} - \sigma(x_1)\\
&\qquad\times[\rho u_i - (\rho u_i)_{\text{target}}]\\
&\partial_t e + U(x_1)(e)_{,1} + (e u_j)_{,j} = (u_j\tau_{ij})_{,j} - \sigma(x_1)(e - e_{\text{target}})
\end{aligned}
\tag{1}
$$

The σ terms drive the solution toward a quiescent target state. This target state may be estimated by solving the flow with $\sigma = 0$, using only the artificial convection aspect of the proposed conditions. Global conservation properties and asymptotic behavior also may be used to estimate the target state. A form is chosen for U and σ such that they either have compact support within the inflow/outflow zones or that they become exponentially small within the physical domain of the problem. Analysis is simplified if the following form is chosen:

$$
\sigma(x_1) = \begin{cases}
\sigma_{o_l}\left(\dfrac{W_{\sigma_l} - x_1}{W_{\sigma_l}}\right)^{\beta_{\sigma_l}} & 0 \le x_1 < W_{\sigma_l}\\
0 & W_{\sigma_l} \le x_1 < X_{1_{\max}} - W_{\sigma_r}\\
\sigma_{o_r}\left[\dfrac{x_1 - (X_{1_{\max}} - W_{\sigma_r})}{W_{\sigma_r}}\right]^{\beta_{\sigma_r}} & X_{1_{\max}} - W_{\sigma_r} \le x_1 \le X_{1_{\max}}
\end{cases}
\tag{2}
$$

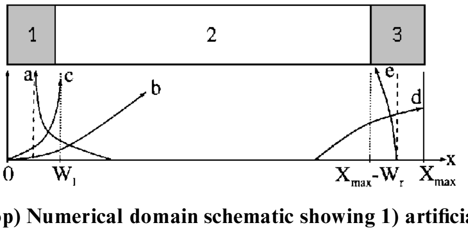

Fig. 1 Top) Numerical domain schematic showing 1) artificial inflow zone, 2) physical computational domain, and 3) artificial outflow zone. Bottom) Characteristics associated with discussion in text: Dotted vertical line (...) is the physical domain edge, and dashed vertical line (– – –) is the sonic point for the artificial convection.

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The equation for $U(x_1)$ is analogous. The domain in x_1 extends from zero to $X_{1_{\max}}$. W_U and W_σ are the widths of the supports for σ or U on the left or right side indicated by l or r subscripts. U_o and σ_o indicate the maximum values that the terms will have at the left or right domain edges. U_o is taken to be greater than the sound speed at each domain edge. This forces the flow to become supersonic and allows us to specify a hard inflow boundary condition.

Analysis of Zones

Direct analysis is impossible for the full equations. We turn to a simplified equation with many of the gross features of the full equations that models a simple shear flow:

$$\partial_t \phi + U_T(x)\partial_x \phi = -\sigma(x)\phi \tag{3}$$

$U_T(x)$ is the total convective velocity. It consists of the artificially imposed velocity $U(x)$ plus any physical convection associated with disturbance modes that we will be considering. We will consider several different U_T corresponding to different situations at the inflow and outflow boundaries; σ is the exponential damping coefficient, ϕ is a field variable that is studied as an outgoing disturbance that we wish to quietly dissipate or as an incoming disturbance that we want to quietly pass into the domain unchanged. These cases are considered by studying the characteristic solutions of Eq. (3).

The discussion is centered about a jet flow. We first consider an acoustic wave traveling toward the inflow in the quiescent fluid exterior to the jet. The path in $x - t$ space of such a disturbance is depicted by characteristic curve a in Fig. 1. For this curve, $U_T(x) = U(x) - c$, where c is the speed of sound. The sound wave will slow as it encounters the artificial convection velocity and eventually will stop. The characteristic asymptotes to the point where $U(x) = c$ and the exponential damping act to annihilate the wave. Because the wave spends a long time in the neighborhood of the sonic point, the damping need not be large.

Next we consider the turbulence in the jet shear layer to estimate the effect that the inflow zone has upon it. We invoke Taylor's hypothesis in applying Eq. (3) to this situation. Structures in the jet will travel at a convective velocity U_c, and the characteristic path for such a structure is shown as curve b in Fig. 1. The structure is accelerated through the inflow zone by the artificial convection term and then slows, reaching its natural convection velocity outside the zone. The damping of a particular turbulent structure, in this case undesirable, is dependant upon the time spent by the structure in the region of positive σ. To estimate the maximum amount of damping that might occur, we consider a structure with zero convective velocity. The characteristic curve corresponding to this behavior is depicted by c in Fig. 1. We may solve Eq. (3) for this case and obtain the value of ϕ entering the physical domain relative to its initial value ϕ_o. All β exponents are taken to be 3. This forces the first three derivatives of σ and U to be continuous across the interface, and this was found necessary to yield smooth numerical solutions. The analytical solution is

$$\frac{\phi}{\phi_o} = \exp\left(-\frac{\sigma_{o_l}}{W_{\sigma_l}^3} \left\{ (W_{\sigma_l} - W_{U_l})^3 s + \left[6(W_{\sigma_l} - W_{U_l})^2 - \frac{2W_{U_l}^3}{W_{U_l} + 2U_{o_l} s} \right] \frac{W_{U_l}^3}{2U_{o_l}} \right.\right.$$
$$\left.\left. \times \sqrt{\frac{1}{W_{U_l}^2} + \frac{2U_{o_l}}{W_{U_l}^3}} + \frac{3(W_{\sigma_l} - W_{U_l}) \ln\left[(1/W_{U_l}^2) + (2U_{o_l}/W_{U_l}^3)\right] W_{U_l}^3}{2U_o} \right\} \right)_{s=0}^{\left[W_{U_l}/2U_{o_l}(W_{U_l} - W_{\sigma_l})^2\right] - (W_{U_l}/2U_{o_l})} \tag{4}$$

Taking practical values for the parameters $W_{\sigma_l} = r_o$, $W_{U_l} = 2.5r_o$, $U_{o_l} = 1.15c_\infty$ and $\sigma_{o_l} = 0.05c_\infty/r_o$, where r_o is the jet radius (or similar problem length scale) and c_∞ is the speed of sound, we estimate that an incoming turbulence structure is damped by a maximum of 1.5%.

At the outflow, flow structures are accelerated out of the domain by the $U(x)$ term (characteristic d in Fig. 1). Any sound generated in the outflow zone will either be convected out of the computational box or travel against the artificially imposed convection velocity into the domain and thus will be exposed to the positive σ region for some time. The σ need not be small as for the inflow, and a reflected sound wave, such as that indicated by characteristic e in Fig. 1, should be mostly dissipated before it enters the physical domain.

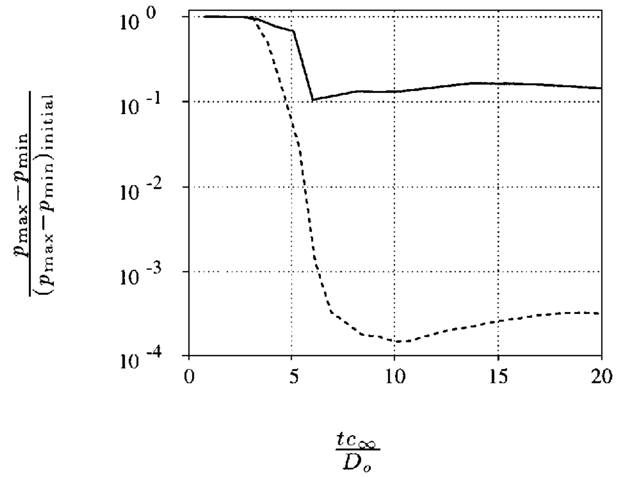

Fig. 2 Maximum pressure difference normalized by initial maximum pressure difference as the vortex passes out of the domain: - - - -, proposed zonal boundary condition; and ——, Thompson boundary condition.

One-dimensional tests of the proposed scheme applied to the full equations showed that the power form for $U(x_1)$ as used in the preceding analysis caused low-amplitude 2δ type disturbances near the boundary between the physical and nonphysical domains. This was remedied using a hyperbolic-tangent-based function that becomes exponentially small within the domain:

$$U(x_1) = \tfrac{1}{2}U_{o_l}[1 + \tanh(-f_l x_1)]$$
$$+ \tfrac{1}{2}U_{o_r}\left\{1 + \tanh\left[f_r(x_1 - X_{1_{\max}})\right]\right\} \tag{5}$$

Performance

The quantification of boundary condition performance is extremely problem dependent, and it is beyond the scope of this Note to do a detailed parametric study of these proposed boundary conditions. We consider two model problems to evaluate the performance of the proposed scheme relative to the local boundary condition of Thompson,[13] which is well known.

To test the outflow boundary, the two-dimensional fully compressible Navier–Stokes equations are solved with a zero circulation vortex[14] as the initial condition. The vortex has $U_{\max}/c_\infty = 0.6$ and is convected with $U_c/c_\infty = 0.75$ through the boundary. The boundary zone was $7.2D_o$ wide, where D_o was the diameter corresponding to the maximum velocity of the vortex. Other boundary parameters were $U_{o_r} = 1.15c_\infty$, $\sigma_{o_r} = 1.125c_\infty/D_o$, and f_r in Eq. (5) was set so that $U(X_{\max} - W_{U_r}) = 0.00015c_\infty$. The other boundaries in the problem were far away and did not affect the results. The nondimensionalized disturbance pressure is plotted in Fig. 2. An ideal boundary condition would have the disturbance pressure become zero as the vortex exits the domain at nondimensional time 5. The zonal boundary condition offers a dramatic improvement over Thompson's condition.

We next test the inflow zone. The linearized Navier–Stokes equations are solved with a packet of plane acoustic waves inclined at

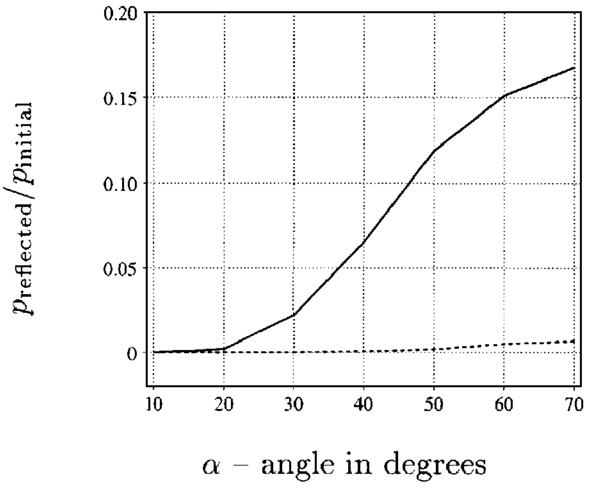

Fig. 3 Normalized pressure perturbation evolution as the vortex passes out of the domain vs angle of incidence on boundary measured from normal: - - - -, proposed zonal boundary condition; and ——, Thompson boundary condition.

an angle to the boundary as the initial condition. The waves propagate toward the inflow boundary. The boundary zone is 2.5λ wide where λ is the acoustic wavelength. Other boundary parameters are $U_{ol} = 1.15c_\infty$, $\sigma_l = 0.035c_\infty/\lambda$, and f_l in Eq. (5) is set so that $U(X_{\max} - W_{Ul}) = 0.01c_\infty$. The transverse boundaries are periodic. Reflection amplitudes are plotted in Fig. 3 as a function of angle. The reflected amplitude would be zero for an ideal boundary condition. The zonal boundary condition is seen to perform significantly better than the Thompson boundary condition, especially at angles away from normal incidence.

Reflections are strongly dependent on zone size, and accuracy may be increased by increasing the size of the zones.

Conclusions

A simple new zonal boundary condition has been proposed. It is based upon the addition of dissipative and convective terms to the compressible Navier–Stokes equations. The scheme has been analyzed using a one-dimensional model equation and validated with two model problems. Boundary reflections are very significantly reduced compared to the local boundary condition of Thompson.

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Diffusion Flame Adjacent to a Rotating Solid Fuel Disk in Zero Gravity

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Introduction

IN the past, a number of studies have examined the characteristics of one-dimensional laminar stagnation-point diffusion flames burning adjacent to solid fuel surfaces. The stagnation flowfield in these past studies has been created by free or forced convection, i.e., by buoyancy effects or by an air jet impinging on the fuel surface (see, for example, Refs. 1 and 2). The focus of the current study is to examine the characteristics of a diffusion flame established adjacent to an infinitely large rotating solid-fuel disk in the absence of buoyancy forces. In this different type of one-dimensional diffusion flame, the flowfield is created by rotation of the fuel disk and the viscosity of the gases next to the disk. For laminar, incompressible, nonreacting flow, this problem is reduced to the well-known von Kármán rotating disk problem.[3] A similarity solution is possible, which is an exact solution of the Navier–Stokes equations. Heat transfer in a compressible flow adjacent to a rotating disk has been treated by Ostrach and Thornton.[4] The first work involving combustion of a rotating disk is by Vedha-Nayagam et al.,[5] in which a diffusion flame analysis has been performed under the assumption of infinitely fast kinetics. In the present work, we extend the analysis to include a finite-rate chemical reaction and surface thermal radiation loss. This enables us to study the question of flame extinction and the flame behavior at low disk rotation rates. The flammability of a rotating subject can be an important consideration in spacecraft fire safety.

Theoretical Model

The combustion model used here assumes the following: a one-step forward gas-phase reaction of second order occurs near the fuel surface, all gases and mixtures of gases follow the ideal gas law, the product of density and viscosity is constant, the diffusion coefficients of all gas species are equal, the specific heats of the gas species are constant and equal, the Prandtl (Pr) and Schmidt (Sc) numbers

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